# Lower Duwamish Waterway Group

Port of Seattle / City of Seattle / King County / The Boeing Company

Appendix A Inverse Distance Weighting Methodology for Interpolating Surface Sediment Chemistry

**Final Feasibility Study** 

Lower Duwamish Waterway Seattle, Washington

## FOR SUBMITTAL TO:

The U.S. Environmental Protection Agency Region 10 Seattle, WA

The Washington State Department of Ecology Northwest Regional Office Bellevue, WA

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Prepared by: **A=COM** 

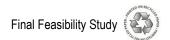
# **Table of Contents**

A.1	Introdu	action	A-1
A.2	Geogra	aphic Framework for Interpolation	A-2
		nterpolation Methodology	
		S	
	A.4.1	Total PCB Results	A-5
	A.4.2	Arsenic Results	A-7
	A.4.3	cPAH Results	A-7
	A.4.4	Dioxins/Furans Interpolation Approach	A-8
A.5		isions	
A.6	Refere	nces	A-9



# **List of Tables**

Table A-1	IDW Parameters for Total PCBs Identified in the 2006 Interpolation Memorandum	A-11
Table A-2	Concentration Ranges Used to Compare Interpolation Results	A-11
Table A-3	Prescreening – Error Statistics for Maximum/Minimum Number of Neighboring Samples Using P5/R150	A-12
Table A-4	Cross-validation Error Statistics for the LDW-wide Interpolation of Total PCBs	A-13
Table A-5	Cross-validation Error Statistics for the Reach-wide Interpolation of Total PCBs	A-14
Table A-6	Summary of LDW-wide Observed False Predictions for Total PCBs by Concentration Range	A-15
Table A-7	Estimated Sediment Surface Areas by Concentration Range for Total PCBs	A-16
Table A-8	Surface Sediment Mapping Methods and Estimated Total PCB SWACs used in the RI/FS Documents	A-17
Table A-9	Cross-validation Error Statistics for the LDW-wide Interpolation of Arsenic	A-18
Table A-10	Summary of LDW-wide Observed False Predictions for Arsenic by Concentration Range	A-19
Table A-11	Estimated Sediment Surface Areas by Concentration Range for Arsenic	A-20
Table A-12	Cross-validation Error Statistics for the LDW-wide Interpolation of cPAHs	A-21
Table A-13	Summary of LDW-wide Observed False Predictions for cPAHs by Concentration Range	A-24
Table A-14	Estimated Sediment Surface Areas by Concentration Range for cPAHs	A-25

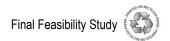


# **List of Figures**

Figure A-1	Comparison of Total PCB Surface Sediment Maps Using Different Interpolation Methods	A-26
Figure A-2	Comparison of Empirical vs. Interpolated Total PCB Concentrations in Surface Sediment	A-27
Figure A-3	Comparison of Arsenic Surface Sediment Maps Using Thiessen Polygons and IDW Interpolation	A-28
Figure A-4	Comparison of Empirical vs. Interpolated Arsenic Concentrations in Surface Sediment	A-29
Figure A-5	Comparison of cPAH Surface Sediment Maps Using Thiessen Polygons and IDW Interpolation	<b>A-</b> 30
Figure A-6	Comparison of Empirical vs. Interpolated cPAH Concentrations in Surface Sediment	A-31

# **List of Attachments**

Attachment A-1 The Inverse Distance Weighting Method



## A.1 Introduction

This appendix documents the methodology used to interpolate concentrations of human health risk-driver contaminants in surface sediments of the Lower Duwamish Waterway (LDW). A draft memorandum on this topic was previously submitted to the U.S. Environmental Protection Agency (EPA) and the Washington State Department of Ecology (Ecology) (RETEC | ENSR 2007). EPA subsequently issued draft comments (EPA 2008), which stated that:

"...the agencies have no major concerns with the chosen methodology and the subsequent updates made to streamline the IDW interpolation and RMSE calculations. The IDW model seems reasonable given the geographic location, the type of data collected, and the ease of interpreting the results."

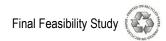
A second memorandum synthesized the first memorandum and was submitted to EPA and Ecology for the purposes of: 1) addressing specific comments from EPA on the inverse distance weighting (IDW) parameterizations for arsenic and total polychlorinated biphenyls (PCBs) presented in the first memo; and 2) providing information for the parameterization of carcinogenic polycyclic aromatic hydrocarbons (cPAHs), which were not interpolated when the first memorandum was submitted (ENSR 2008). A third memorandum describing the use of Thiessen polygons to interpolate surface sediment concentrations of dioxins/furans was submitted to EPA and Ecology for review in early 2010 (AECOM 2010), after collection of additional dioxin/furan surface sediment data in 2009 and 2010. This appendix summarizes the data analysis and findings presented in these three memoranda.

Early in the feasibility study (FS) process, the IDW interpolation of LDW surface sediment total PCB concentrations was developed in consultation with EPA and Ecology (Windward 2006; RETEC 2006). The interpolation method was described in the document *Technical Memorandum: GIS Interpolation of Total PCBs in LDW Surface Sediment* (Windward 2006; hereinafter referred to as the 2006 interpolation memo). The Lower Duwamish Group (LDWG) streamlined the interpolation method from that described in the 2006 interpolation memorandum to better support application of the bed composition model (BCM; RETEC 2007). Specifically, the streamlined method enables interpolation over the entire LDW in a single computational step, as opposed to requiring separate interpolations within each reach, as was done previously and documented in the 2006 interpolation memo. This modification eliminated the additional manipulations previously required to reconcile interpolated results in areas where the three reaches overlap.

The risk drivers parameterized for IDW interpolation are total PCBs, arsenic, and cPAHs. The possibility of using IDW to interpolate the dioxin/furan concentrations was

An explanation of the IDW method is provided in Attachment A-1.





investigated, but sufficient data coverage was not available over the study area to adequately parameterize for IDW (AECOM 2010).

The Sediment Management Standards (SMS) contaminants are evaluated as individual points rather than being spatially interpolated (and Thiessen polygons were used to determine the extent of point exceedances). The IDW interpolation analysis discussed in this appendix was completed a couple of years ago and therefore used the remedial investigation (RI) baseline surface sediment dataset. The RI baseline dataset represented the data available when these evaluations were conducted (RETEC | ENSR 2007). The FS uses the FS baseline dataset, which includes more recent data, and the same parameterization methods summarized in this appendix. Because of the inclusion of newer data, the interpolated maps based on the FS baseline dataset that are used in this FS differ slightly from those included in this appendix. The spatially-weighted average concentrations (SWACs) associated with the more recent interpolations used in this FS are also slightly different from those reported in this appendix.

## A.2 Geographic Framework for Interpolation

The FS uses a single-step interpolation over the entire study area compared to the methods described in the 2006 memo, where interpolation was applied separately for the three reaches (Windward 2006) (Table A-1).<sup>2</sup> This has the following advantages:

- Eliminates the multiple computations previously required to accommodate the merging of interpolated values for the three separate reaches, an important time-saving benefit, especially if post-remediation scenarios of the BCM involve re-interpolation following insertion of post-remedy bed sediment replacement values.
- ♦ Allows calculation of the cross-validation root mean square error (CV-RMSE) in the 0.4 river mile (RM) of overlaps (i.e., between the north and middle reaches and between the middle and south reaches) that were excluded from CV-RMSE calculation previously.
- Eliminates error introduced in the 0.4-RM overlaps created by the averaging function. The averaging function gives more weight to the premosaic value in a

The three reaches defined in the 2006 memorandum allowed the IDW to account for differences in the orientation of the waterway. A mosaic function was used to merge the three reaches into a single layer with 0.4 mile of overlap between merged areas. The mosaic function combines two or more overlapping grid cells into a single output. The mosaic function used in the 2006 interpolation memorandum used the Hermite cubic proximity algorithm, which incorporates the overlap width and distance of each grid cell from the overlap edge to calculate a weighted mean of the overlapping grid-cell values.





cell than would be accorded if the cell was initially influenced by all available data points, as is the case in an LDW-wide interpolation scheme.

The LDW study area encompasses 441 acres from RM 0 to RM 5.0. This area includes an additional 10 acres not included in the RI. These additional 10 acres were added as the result of revisions to shoreline, under-pier areas, and top-of-bank delineation.

## A.3 IDW Interpolation Methodology

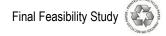
The LDW-wide IDW methodology is similar to that described previously (Windward 2006), except that the geographic template spans the entire study area instead of being segmented into three reaches. Previously, each reach of the river was mapped using an ellipse with a reach-specific search radius and directionality oriented to the flow of the river. To map the LDW as a single unit, a common ellipse with no angle (circle) is required. This reduces the dimensional element of parameterization to determine an appropriate search radius. Therefore, optimization of the interpolation parameters focuses on the exponential power, search radius, and maximum/minimum number of closest samples (nearest neighbors) used to interpolate grid-cell concentrations, using the following systematic approach:<sup>3</sup>

- ◆ Vary the exponential power (1 to 10), search radius (75 to 250 feet [ft]), and the maximum/minimum number of neighboring samples (1 to 10), then interpolate grid-cell concentrations.
- Use the CV-RMSE, the observed RMSE, and the number of false predictions to identify a "common" set of interpolation parameters that yield the lowest error.
- Generate an LDW-wide sediment concentration map by IDW interpolation using the "common" set of parameters. Calculate and compare the number of acres predicted to fall within specified concentration ranges, and compare false positive/false negative predictions.

Two variants of RMSE statistics were used to optimize the IDW interpolation parameters. First, the "observed RMSE" compares differences between each empirical data point and its underlying interpolated grid-cell value. Second, the CV-RMSE compares the same metric but generates sequential interpolations (one for each data point) by removing that co-located data point prior to interpolation. Thus, cross-validation excludes the empirical concentration from the interpolation dataset and then compares the empirical and interpolated concentrations. CV-RMSE gauges interpolative

Total PCBs were the only risk driver parameterized on a reach-specific basis in the 2006 interpolation memo. Therefore, as an additional point of comparison, this memorandum presents results of applying the selected total PCB (i.e., LDW-wide) parameters and the 2006 interpolation memorandum parameters to the three reaches using the complete RI surface sediment dataset.





sensitivity to variability in the dataset. As dataset variability increases, so too does the CV-RMSE.

Interpolative accuracy was also evaluated by comparing the frequency of false predictions. The false prediction frequency varies directly with the RMSE. For this comparison, the types and numbers of false predictions were counted relative to concentration ranges.<sup>4</sup> The concentration ranges identified for total PCBs, arsenic, and cPAHs are shown in Table A-2. The ranges span the generally anticipated magnitude of natural and area background concentrations (as they were understood at the time in the initial FS process) and risk-based threshold concentrations. The ranges for total PCBs and arsenic also span the SMS sediment quality standard (SQS) and cleanup screening level (CSL) values.<sup>5</sup>

Interpolation yields a "false" prediction when the interpolated sample concentration at a specific location falls within a higher (false positive) or lower (false negative) concentration range than the empirical data point. For example, an interpolated total PCB concentration of 65 micrograms per kilogram dry weight ( $\mu$ g/kg dw) co-located with an empirical value falling in the range of 0 to 60  $\mu$ g/kg dw is termed a "false positive." Similarly, an interpolated total PCB concentration of 300  $\mu$ g/kg dw co-located with an empirical value falling in the range of 720 to 1,300  $\mu$ g/kg dw is termed a "false negative."

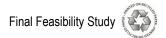
## A.4 Results

This section presents the cross-validation error statistics, false negative/positive predictions, and estimated IDW SWACs by concentration range for total PCBs, arsenic, and cPAHs.

Before running these analyses, the "nearest neighbor" parameter (i.e., the maximum and minimum number of samples within the search radius (R) that can be used in the interpolation) was prescreened to determine its sensitivity on the IDW interpolation. Error statistics for total PCBs, arsenic, and cPAHs were calculated using a power (P) of 5 and a circular search radius of 150 ft (P5/R150). This power and search radius combination falls in the middle of the ranges tested. Table A-3 shows that the CV-RMSE

<sup>&</sup>lt;sup>5</sup> For total PCBs, this comparison assumes an average total organic carbon content of 1.9% for surface sediment.





<sup>&</sup>lt;sup>4</sup> The magnitude of error between predicted and actual concentrations was considered less important than whether or not the predicted value is above or below some risk- or feasibility-based threshold. For example, there is a two-fold difference between a predicted cPAH value of 2,000 and an actual value of 4,000, but both cPAH values are well above the 10-6 risk-based threshold concentration for site-wide netfishing (380 μg toxic equivalent (TEQ)/kg dw). For this reason, concentration ranges provide better context for parameter evaluation than point value comparisons.

and RMSE are relatively insensitive to the maximum and minimum number of samples within the range tested. A fixed maximum/minimum value of 10/1 was selected for evaluating optimal search radius and power parameters for total PCBs, arsenic, and cPAHs, as described below.

### A.4.1 Total PCB Results

Results of the reach-by-reach parameter evaluation for total PCBs are presented in Table A-4. Within each reach, changes in CV-calculated mean absolute error as a function of power are statistically insignificant (by t-test). However, the observed-RMSE declines sharply in the vicinity of an exponential power of 3 to 5, suggesting that an appropriate LDW-wide set of parameters lies within that range. The decline in observed-RMSE with power reflects how this parameter influences the interpolation to more closely mirror the measured dataset.

LDW-wide IDW parameters evaluated for total PCBs and their associated error statistics are also shown in Table A-4. Combinations of powers from 1 through 10 were tested while varying the search radii between 75 ft and 250 ft. The influence of search radius on the CV-RMSE was very limited. In all cases, the CV-RMSE ranged between approximately 7,800 and 9,660. Within each search radius grouping, the CV-calculated mean absolute errors were statistically indistinguishable across the range of power values, as determined by non-parametric analysis of variance<sup>6</sup> (Kruskal-Wallis test;  $\alpha = 0.05$ ; p-values ranged from 0.396 to 0.916). Thus, no statistical difference exists between the calculated CV-RMSEs within each search radius grouping.

A power of 5 and a circular search radius of 150 ft (P5/R150) were selected for further scrutiny on an LDW-wide basis. A circular search radius of 150 ft was selected because it aligns well with the spatial scales of the river and sample point distribution. Also, as discussed above, interpolation results are insensitive to radii in the range tested. Further, this search radius is consistent with the 2006 interpolation memo, which recommended an ellipse with a major axis of 150 ft and a minor axis of 75 ft. A power of 5 was selected because the observed RMSE in the reach-by-reach analysis was lowest in the vicinity of 5.

Table A-5 compares RMSE results for total PCBs obtained using the P5/R150 parameter set relative to those calculated using the 2006 interpolation memorandum parameters. The results are grouped to enable a reach-by-reach comparison of the methods. In

Normality of the dataset was assessed prior to statistical testing. Given the number of samples in the dataset, the Kolmogorov-Smirnov normality test was used rather than the Shapiro-Wilk test, which is restricted to datasets with 50 or fewer samples. Natural log (LN)-transformation of the data improved normality, but results of the Kolmogorov-Smirnov normality test indicated that transformed data were still not normal (p-values = 0.000). Thus, the non-parametric Kruskal-Wallis test was used for statistical testing rather than analysis of variance.





addition, LDW-wide summary statistics are provided. The reach-by-reach comparison shows method comparability on a CV-RMSE basis. Indeed, the reach-specific CVcalculated absolute errors generated by the two methods are not statistically different (Mann-Whitney test<sup>7</sup>;  $\alpha = 0.05$ ; p-values ranged from 0.238 to 0.486). The observed RMSE values using P5/R150 are considerably lower. This reflects the influence of the power parameter on the degree to which the distance of a measured data point from a grid-cell location affects the interpolation. On an LDW-wide basis, the calculated CV-RMSE values using P5/R150 are within the range of values established reach-by-reach using the 2006 interpolation memorandum parameters (Table A-5). The observed LDWwide RMSE values generated by the P5/R150 interpolation are generally lower than the reach-by-reach values resulting from the 2006 interpolation memorandum parameters. This demonstrates that the P5/R150 predictions better mirror the actual dataset, an observation that is further illustrated in Table A-6, which compares the prediction accuracy for total PCBs (measured versus predicted) on reach-by-reach and LDW-wide bases. The number of stations correctly assigned to concentration ranges is appreciably higher by LDW-wide interpolation using P5/R150.

Table A-7 presents the calculated surface areas of sediment (in acres) grouped by total PCB concentration range. The reach-by-reach and LDW-wide interpolation methods estimate roughly the same numbers of affected acres above and below a total PCB concentration of 240  $\mu$ g/kg dw. Above 240  $\mu$ g/kg dw, the number of estimated acres in each total PCB concentration range is essentially equivalent between the two methods. Table A-7 also compares the SWACs for each method. The differential in SWAC values (360  $\mu$ g/kg dw vs. 375  $\mu$ g/kg dw) is only 4%. When taken in conjunction with the cross-validation results, the LDW-wide P5/R150 interpolation provides equivalent, if not better prediction accuracy, than the 2006 interpolation memorandum parameters.

The SWAC values obtained using LDW-wide IDW interpolation parameters compare favorably to the corresponding results obtained in the RI, which used different methods (Table A-8). In some cases, the area interpolated varied (e.g., variable river miles) depending upon the intended use of the specific statistics. Overall, the results of the various methods used to date and the proposed IDW parameterization compare very well.



Normality of the dataset was assessed prior to statistical testing. Given the number of samples in the dataset, the Kolmogorov-Smirnov normality test was used rather than the Shapiro-Wilk test, which is restricted to datasets with 50 or fewer samples. LN-transformation of the data improved normality, but results of the Kolmogorov-Smirnov normality test indicated that transformed data were still not normal (p-values < 0.048). Thus, the non-parametric Mann-Whitney test was used rather than a t-test.

<sup>8</sup> When an error in estimation occurs, the model tends to overpredict (i.e., gives a false positive).

This dry weight concentration value is the approximate equivalent of the SQS for total PCBs [12 mg/kg organic carbon] assuming an average total organic carbon [TOC] value for LDW surface sediment of 1.9%.

Based on this analysis, the recommended IDW parameters for interpolating total PCBs in this FS are a circular search radius of 150 ft and an exponential power of 5. These parameters yield results comparable to interpolated conditions mapped using the 2006 interpolation memorandum (reach-by-reach) IDW parameters (Figure A-1). The empirical (measured) data points are superimposed on the FS IDW interpolation (Figure A-2), which enables a qualitative comparison of interpolative accuracy.

### A.4.2 Arsenic Results

For arsenic, the CV-RMSE and observed RMSE values were first calculated on a reach-by-reach basis assuming fixed values for search radius (500 ft) and the maximum/ minimum number of samples (10/1). The results show that the error statistics are insensitive to power above a value of approximately 3 (Table A-9). Combinations of powers from 1 through 5 and 10 were tested while varying the search radii between 75 ft and 250 ft. The CV-RMSE and observed RMSE values from LDW-wide interpolation are insensitive to power above a value of approximately 3 (Table A-9). Within each search radius grouping, the CV-calculated absolute errors were statistically indistinguishable across the range of power values, as determined by non-parametric analysis of variance (Kruskal-Wallis test;  $\alpha$  = 0.05; p-values ranged from 0.975 to 0.998). The influence of search radius on the CV-RMSE was also small. In all cases, the CV-RMSE ranged between 55 and 70.

Table A-10 compares the frequency of false predictions for the following power and search radius combinations: P3/R150, P3/R250, P5/R150, and P5/R250. The lowest number of false predictions occurs using the P5/R150 combination. Table A-11 presents sediment surface areas and calculated SWAC values using the different power and search radius combinations. Again, the results show a general lack of sensitivity to the specific parameters used. For example, the SWAC ranges from 15 to 16 mg/kg dw. Given this lack of sensitivity and the slightly lower level of false predictions, P5/R150 is a statistically justifiable and reasonable set of parameters for arsenic.

Figure A-3 shows that the arsenic surface sediment concentration maps generated by IDW (P5/R150) and Thiessen polygons are very similar. Figure A-4 shows the arsenic surface sediment concentration map generated by IDW (P5/R150) with the sample data points superimposed for comparison. Arsenic was not evaluated with as much rigor as total PCBs nor compared to any RI methods because the RI only interpolated total PCBs with IDW.

### A.4.3 cPAH Results

Observed RMSE and CV-RMSE statistics were calculated for several combinations of exponential power (range: 1 to 10) and search radii (range: 75 ft to 250 ft) on an LDW-wide basis (Table A-12). At all search radii, the observed RMSE values trend from highest to lowest as the exponential power is increased from 1 to 10. This is an expected





outcome because increasing power correspondingly increases the influence of any given observed point concentration on the interpolated value at that location. At a particular power, the observed RMSE values are essentially identical regardless of search radius. Similarly, the results in Table A-12 show that the declining trends in observed RMSE flatten out and remain approximately constant above a power of 5. The relatively high CV-RMSE values shown in Table A-12 compared to the observed RMSE value are a reflection of variability in the dataset.

Within each search radius grouping, the CV-calculated absolute errors were statistically indistinguishable across the range of power values, as determined by a non-parametric analysis of variance (Kruskal-Wallis test;  $\alpha$  = 0.05; p-values ranged from 0.997 to 1.000). Thus, there is no statistical difference between the calculated CV-RMSEs within each search radius grouping.

Table A-13 compares the frequency of false predictions for the following power and search radius combinations: P5/R150, P5/R175, P6/R150, and P6/R175. These combinations were selected for further consideration because they represent a region of the parameter continuum where the observed RMSE values are low (and constant), and the CV-RMSE values are mid-range. The fewest false predictions occur using the P6/R150 or P6/R175 combinations.

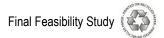
Table A-14 presents sediment surface areas (within each concentration range) and calculated SWAC values using the different power and search radius combinations. The results show a lack of sensitivity within this grouping of parameters. For example, the different SWAC values vary by no more than 2  $\mu$ g TEQ/kg dw. Based on the foregoing analysis, P6/R150 is a statistically justifiable and reasonable set of parameters for interpolating cPAH by IDW and is used in this FS.

Figure A-5 shows the cPAH surface sediment concentration map generated by IDW (P6/R150) with the empirical data points superimposed on the map for comparison.

## A.4.4 Dioxins/Furans Interpolation Approach

Data for dioxins/furans are not as numerous as for other risk-driver contaminants. For this reason, the Thiessen polygon method was selected for use in the FS. In a memorandum prepared and submitted to EPA and Ecology in March 2010, IDW and Thiessen polygon interpolation methods were explored. Based on the lack of change observed in the SWAC (between the IDW and Thiessen polygons) and on visual inspection of the maps, Thiessen polygons are considered adequate for the spatial characterization of dioxin/furan concentrations. However, polygons that extend from one bank to another (across the navigation channel) should be used with caution because it has been observed that concentrations upstream and downstream of a given location have greater similarity than those in a cross-channel direction.





## A.5 Conclusions

Interpolation parameters suitable for IDW-interpolation of LDW surface sediment chemistry data were developed for total PCBs, arsenic, and cPAHs. These parameters are applied on an LDW-wide geographic framework as opposed to three separate reaches as was done previously (Windward 2006). As a result of these analyses, the following input parameters were used in the FS:

- ♦ Power of 5, maximum/minimum nearest neighbors 10/1, circular search radius 150 for total PCBs
- ♦ Power of 5, maximum/minimum nearest neighbors 10/1, circular search radius 150 for arsenic
- ♦ Power of 6, maximum/minimum nearest neighbors 10/1, circular search radius 150 for cPAHs.

These parameters were selected because they represent the best-optimized parameters from the cross-validation results. For dioxins/furans, Thiessen polygons were used in the FS for mapping because of the smaller dataset.

## A.6 References

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Table A-1 IDW Parameters for Total PCBs Identified in the 2006 Interpolation Memorandum

LDW Study Area Reach	Power <sup>a</sup>	Search Radius (ft) and Angle <sup>b,c</sup>	Search Radius Shape <sup>d</sup>	Maximum/Minimum Nearest Neighbors <sup>e</sup>
North	1	150 x 75, 0	cross-axis quadrants	2/1
Middle	1	150 x 75, 300	axis quadrants	6/2
South	1	150 x 150, 300	axis quadrants	4/4

Source: 2006 Interpolation Memo (Windward 2006).

#### Notes:

- a. Power: The weighting parameter applied to the interpolation. As the power increases, the weighting of a sample result at distance from the sample location diminishes.
- b. Search Radius Shape: The division of the search shape (circle/ellipse) into quadrants and the orientation of those quadrants.
- c. Angle: The orientation of the search radius relative to north (north=0/360, south=180).
- d. Search Radius Shape (Major/Minor Axis): The length (in ft.) of the axes of an ellipse, major being the longer of the two.
- e. Maximum/Minimum Nearest Neighbors: The maximum and minimum number of closest samples used to interpolate a grid cell.

ft = feet; IDW = inverse distance weighting; LDW = Lower Duwamish Waterway; PCB = polychlorinated biphenyl

Table A-2 Concentration Ranges Used to Compare Interpolation Results

Total PCB Concentration Range (μg/kg dw)	Arsenic Concentration Range (mg/kg dw)	cPAH Concentration Range (μg TEQ/kg dw)
≤ 60	≤12	≤90
>60-120	>12-16	>90-150
>120-240	>16-20	>150-380
>240-480	>20-57ª	>380-900
>480-720	>57-93 <sup>b</sup>	>900
>720-1,300	>93 <sup>b</sup>	
>1,300		

### Notes:

- a. The SMS sediment quality standard value for arsenic is 57 mg/kg dw.
- b. The SMS cleanup screening level value for arsenic is 93 mg/kg dw.

cPAH = carcinogenic polycyclic aromatic hydrocarbon; mg/kg dw = milligrams per kilogram dry weight; µg/kg dw = micrograms per kilogram dry weight; PCB = polychlorinated biphenyl; SMS = Sediment Management Standards; TEQ = toxic equivalent

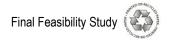


Table A-3 Prescreening – Error Statistics for Maximum/Minimum Number of Neighboring Samples Using P5/R150

	Maximum Number of Neighboring Samples						
Parameter	1	2	4	6	8	10	
Total PCBs P5/R150	-	-			-		
CV-RMSE	10300	9200	9100	9200	9100	9100	
Observed RMSE	290	250	260	260	280	260	
Total False Positive/Negative	6	13	13	13	14	14	
Arsenic P5/R150							
CV-RMSE	70	69	69	69	69	69	
Observed RMSE	1	0	0	0	0	0	
Total False Positive/Negative	2	5	5	5	5	5	
cPAH P5/R150							
CV-RMSE	840	770	740	740	740	740	
Observed RMSE	23	26	27	27	27	27	
Total False Positive/Negative	3	11	11	11	11	11	

- 1. P5/R150 Power of 5 and circular search radius of 150 feet.
- CV-RMSE and observed RMSE units are micrograms per kilograms dry weight (μg/kg dw) for total PCBs, μg TEQ/kg dw for cPAH, and milligrams per kilograms dry weight (mg/kg dw) for arsenic.
- 3. Minimum number of neighboring samples is 1 in all cases.

cPAH = carcinogenic polycyclic aromatic hydrocarbon; CV-RMSE = cross-validation root mean square error; P = power; PCB = polychlorinated biphenyl; R = radius; RMSE = room mean square error; TEQ = toxic equivalent



Table A-4 Cross-validation Error Statistics for the LDW-wide Interpolation of Total PCBs

			Cross-Validation		
Circular Search Radius (ft)	Power	Mean Error	Mean Absolute Error	RMSE	Observed RMSE
North Reach		<u> </u>			
	1	41	523	2,887	622
	2	41	522	2,883	1,043
050	3	5	503	2,895	152
250	4	-33	495	2,956	11
	5	-20	494	2,926	32
	10	-37	500	2,972	11
450	5	-37	510	2,978	27
150	10	-51	510	3,012	10
Mid Reach					
	1	131	1,835	5,815	3,271
	2	148	1,860	5,983	1,418
250	3	142	1,896	6,204	878
	5	132	1,959	6,563	841
	10	139	2,050	7,149	865
150	5	130	1,963	6,567	382
150	10	139	2,053	7,153	865
South Reach					
	1	330	2,494	12,070	6,075
	2	326	2,475	12,880	3,224
250	3	306	2,466	13,510	2,247
	5	280	2,481	14,160	1,185
	10	252	2,508	15,010	403
150	5	287	2,579	14,357	394
130	10	260	2,573	15,240	431
LDW-wide PCBs					
	1	145	1,415	7,829	3,584
	2	136	1,411	8,330	1,040
250	3	119	1,418	8,722	390
200	4	107	1,429	8,961	273
	5	100	1,440	9,131	255
	10	83	1,468	9,656	275
	1	125	1,412	7,831	3,583
	2	124	1,412	8,330	1,040
150	3	113	1,422	8,724	390
150	4	105	1,434	8,963	273
	5	99	1,444	9,132	255
	10	84	1,480	9,657	275



Table A-4 Cross-validation Error Statistics for the LDW-wide Interpolation of Total PCBs (continued)

Circular Search Radius (ft)	Power	Mean Error	Mean Absolute Error	RMSE	Observed RMSE
LDW-wide PCBs (continued)					
	1	53	1,371	8,019	3,403
	2	82	1,404	8,448	1,027
75	3	88	1,424	8,771	389
75	4	86	1,437	8,986	273
	5	83	1,445	9,146	255
	10	71	1,464	9,659	275

- 1. A maximum of 10 and minimum of 1 "nearest neighbor" data points were used in all interpolations.
- 2. Results are insensitive to a power beyond 5. Results using a power of 10 are provided as an outer bound reference point.

ft = feet; LDW = Lower Duwamish Waterway; PCB = polychlorinated biphenyl; RMSE = root mean square error

Table A-5 Cross-validation Error Statistics for the Reach-wide Interpolation of Total PCBs

Reach and Interpolation Method	Count	Mean Error	Mean Absolute Error	Cross-Validation RMSE	Observed RMSE		
2006 Interpolation Memo Method <sup>a</sup>	2006 Interpolation Memo Method <sup>a</sup>						
North Reach	416	-8	455	2,770	820		
Middle Reach	583	56	1,753	5,714	3,479		
South Reach	505	223	2,459	12,298	6,593		
P5/R150 b							
North Reach	416	-37	509	2,978	27		
Middle Reach	583	129	1,963	6,567	382		
South Reach	505	284	2,550	9,132	394		
LDW-wide	1,327	99	1,444	9,132	255		

#### Notes:

- a. Values were generated using the reach-specific parameters in Table A-1 (i.e., data from 2006 Interpolation Memo [Windward 2006]).
- b. Results were obtained by interpolating within the individual reaches using the P5/R150 parameters.

LDW = Lower Duwamish Waterway; P = power; PCB = polychlorinated biphenyl; R = radius; RMSE = root mean square error



Table A-6 Summary of LDW-wide Observed False Predictions for Total PCBs by Concentration Range

PCB Range	2006 Interpolation	n Memo Parameters <sup>a</sup>	P5/	P5/R150		
(µg/kg dw)	False Positives	False Negatives	False Positives	False Negatives		
North Reach						
≤ 60	17	0	2	0		
>60-120	27	0	7	0		
>120-240	11	6	1	0		
>240-480	9	3	1	0		
>480-720	1	4	0	0		
>720-1,300	0	5	0	2		
>1,300	0	1	0	0		
Subtotal	65	19	11	2		
Middle Reach						
≤60	23	0	2	0		
>60-120	39	0	6	0		
>120-240	44	3	5	0		
>240-480	39	2	4	1		
>480-720	14	4	1	0		
>720-1300	19	4	3	0		
>1,300	0	2	0	0		
Subtotal	178	15	21	1		
South Reach						
≤60	33	0	6	0		
>60-120	20	4	0	0		
>120-240	32	5	4	0		
>240-480	27	4	2	0		
>480-720	9	1	1	0		
>720-1,300	13	3	1	0		
>1,300	0	1	0	0		
Subtotal	134	18	14	0		
Grand Total	377	52	46	3		
LDW-wide PCBs						
≤60	62	0	2	0		
>60-120	81	7	1	0		
>120-240	61	17	2	0		
>240-480	51	14	2	1		
>480-720	18	10	2	0		
>720-1,300	20	17	1	2		
>1,300	0	3	0	0		
Grand Total	293	68	10	3		

Source: 2006 Interpolation Memo (Windward 2006).

Notes:

LDW = Lower Duwamish Waterway; µg/kg dw = micrograms per kilogram dry weight; P = power; PCB = polychlorinated biphenyl; R = radius



a. Prediction accuracy is based on comparing the interpolated station value to the concentration range that the measured value falls within.

Table A-7 Estimated Sediment Surface Areas by Concentration Range for Total PCBs

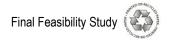
Total PCB	2006 Interpola	tion Memo Parameters	P5/R150		
Concentration Range (µg/kg dw)	Area (acres)	Cumulative Area (acres)	Area (acres)	Cumulative Area (acres)	
≤60	72	72	106	106	
>60-120	106	178	106	212	
>120-240	153	331	118	330	
>240-480	55	386	59	388	
>480-720	13	399	16	404	
>720-1,300	11	410	10	414	
>1,300	19	430	16	430	
Total Area of LDW	430		430		

**SWAC (μg/kg dw)** 375 360

Source: 2006 Interpolation Memo (Windward 2006).

#### Notes:

LDW = Lower Duwamish Waterway;  $\mu g/kg dw = micrograms per kilogram dry weight; P = power; PCB = polychlorinated biphenyl; R = radius; SWAC = spatially-weighted average concentration$ 



<sup>1.</sup> Total LDW surface area was 430 acres when this analysis was conducted.

Table A-8 Surface Sediment Mapping Methods and Estimated Total PCB SWACs used in the RI/FS Documents

Document and Table or Section	Total PCB Concentration	Unit	Measure	Method	River Mile	Dataset <sup>b</sup>	Use
Food Web Model (FWM) a							
Table E.4-1	380	μg/kg dw	SWAC	IDW – 2006 Memo	0.0 - 5.25	RI Baseline	FWM calibration
Remedial Investigation (RI) <sup>a</sup>	Remedial Investigation (RI) <sup>a</sup>						
Section 5.2.1	370	μg/kg dw	SWAC	IDW – 2006 Memo	0.0 - 6.0	RI Baseline	Nature and extent
Comparison of Various Approaches							
	380	μg/kg dw	SWAC	IDW – 2006 Memo	0.0 - 5.0	RI Baseline	
Whole-river SWAC	360	μg/kg dw	SWAC	IDW – Proposed P5	0.0 - 5.0	RI Baseline	Base conditions for application of the BCM
	350	μg/kg dw	SWAC	Thiessen polygon	0.0 - 5.0	RI Baseline	,,

BCM = bed composition model; FS = feasibility study; FWM = food web model; IDW = inverse distance weighting; LDW = Lower Duwamish Waterway; µg/kg dw = micrograms per kilogram dry weight; RI = remedial investigation; P = power; PCB = polychlorinated biphenyl; SWAC = spatially-weighted average concentration





a. The 2006 interpolation memo method was used for the SWAC calculations.

b. This analysis was conducted in 2007 at initiation of the FS; therefore, the RI baseline dataset was used in the analysis.

Table A-9 Cross-validation Error Statistics for the LDW-wide Interpolation of Arsenic

Circular Search			Cross-Validation		
Radius (ft)	Power	Mean Error	Mean Absolute Error	RMSE	Observed RMSE
North Reach Arsenie	C				
	1	1.3	16.0	64	14
	2	0.9	15.7	66	6
500	3	0.1	15.4	67	3
500	4	-0.5	15.4	67	1
	5	-1.0	15.4	67	1
	10	-1.7	15.4	67	0
Mid Reach Arsenic					
	1	0.0	3.4	6	2
	2	0.1	3.6	6	1
500	3	0.2	3.7	6	1
500	4	0.2	3.8	6	1
	5	0.2	3.9	6	1
	10	0.2	4.1	7	1
South Reach Arseni	ic				
	1	-1.9	9.3	61	20
	2	-2.2	8.8	60	2
500	3	-2.1	8.9	61	1
500	4	-1.8	9.2	64	1
	5	-1.4	9.5	67	1
	10	-0.5	10.8	80	1
LDW-wide Arsenic					
	1	-0.8	9.9	55	8
	2	-0.9	9.9	55	1
250	3	-1.0	10.1	56	1
250	4	-1.1	10.3	57	1
	5	-1.0	10.5	59	0
	10	-0.9	11.2	65	1
	1	-0.3	11.4	69	6
	2	-0.6	11.2	68	1
150	3	-0.8	11.4	68	1
150	4	-1.0	11.5	69	1
	5	-1.1	11.6	69	1
	10	-1.2	11.8	69	1
	1	-1.3	11.7	70	5
	2	-1.3	11.8	70	1
75	3	-1.3	11.8	69	1
	4	-1.3	11.8	69	1
	5	-1.3	11.8	70	0

- 1. A maximum of 10 and minimum of 1 "nearest neighbor" data points were used in all interpolations.
- 2. Results are insensitive to a power beyond 5. Results using a power of 10 are provided as an outer bound reference point.

ft = feet; LDW = Lower Duwamish Waterway; RMSE = root mean square error



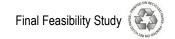


Table A-10 Summary of LDW-wide Observed False Predictions for Arsenic by Concentration Range

	P3/R250 <sup>a</sup>		P5/R250		P3/R150		P5/R150	
Arsenic Range (mg/kg dw)	False Positives	False Negatives	False Positives	False Negatives	False Positives	False Negatives	False Positives	False Negatives
≤12	23	0	11	0	20	0	10	0
>12-16	4	0	2	0	3	0	2	0
>16-20	1	2	1	1	1	2	1	1
>20-57	0	1	0	1	0	1	0	1
>57-93	0	0	0	0	0	0	0	0
>93	0	0	0	0	0	0	0	0
Subtotal	28	3	14	2	24	3	13	2
Total False Predictions	;	31	1	16	2	7	1	5

LDW = Lower Duwamish Waterway; mg/kg dw = milligrams per kilogram dry weight; P = power; R = circular search radius





a. Power and search radius parameter combination (e.g., P5/R150 = power of 5 and circular search radius of 150 feet)

Table A-11 Estimated Sediment Surface Areas by Concentration Range for Arsenic

	IDW Interpolation Parameters									
	P5/R250a		P3/R250		P5/I	P5/R150		R150		
Arsenic Concentration Range (mg/kg dw)	Area (Acres)	Cumulative Area (Acres)	Area (Acres)	Cumulative Area (Acres)	Area (Acres)	Cumulative Area (Acres)	Area (Acres)	Cumulative Area (Acres)		
≤12	254	254	250	250	261	261	259	259		
>12-16	111	365	115	365	103	365	105	364		
>16-20	30	395	29	394	31	396	31	395		
>20-57	29	424	29	423	29	425	29	424		
>57-93	2	426	2	425	1	426	2	426		
>93	4	430	5	430	4	430	4	430		

SWA	16	16	15	15

- 1. Total LDW surface area was 430 acres when this analysis was conducted.
- a. Power and search radius parameter combination (e.g., P5/R150 = power of 5 and circular search radius of 150 feet.)

IDW = inverse distance weighting; LDW = Lower Duwamish Waterway; mg/kg dw = milligrams per kilogram dry weight; P = power; R = circular search radius; SWAC = spatially-weighted average concentration



Table A-12 Cross-validation Error Statistics for the LDW-wide Interpolation of cPAHs

Circular Search	ular Search Cross-Validation					
Radius (ft)	Power	Mean Errora	Mean Absolute Errorb	RMSE	Observed RMSE	
	1	5.5	311	645	180	
	2	11.1	317	668	63	
	3	11.7	326	697	44	
	4	10.3	333	719	33	
250	5	8.8	338	737	27	
250	6	7.8	342	752	24	
	7	7.3	345	765	23	
	8	7.1	347	776	23	
	9	7.2	350	786	22	
	10	7.4	352	794	23	
	1	5.5	310	648	179	
	2	10.7	318	672	63	
	3	11.1	327	699	44	
	4	9.7	334	720	33	
005	5	8.4	339	738	27	
225	6	7.4	342	752	24	
	7	7.0	345	765	23	
	8	6.9	348	776	23	
	9	6.9	350	786	22	
	10	7.1	352	794	23	
	1	4.5	311	651	179	
	2	10.0	319	674	63	
	3	10.4	327	700	44	
	4	9.1	333	721	33	
000	5	7.7	338	738	27	
200	6	6.8	342	753	24	
	7	6.3	345	765	23	
	8	6.2	348	776	23	
	9	6.3	350	786	22	
	10	6.5	352	794	23	
	1	1.4	311	655	172	
175	2	7.0	320	679	63	
	3	7.6	329	703	44	

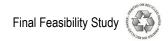


Table A-12 Cross-validation Error Statistics for the LDW-wide Interpolation of cPAHs (continued)

Circular Search	ular Search Cross-Validation					
Radius (ft)	Power	Mean Errora	Mean Absolute Errorb	RMSE	Observed RMSE	
	4	6.6	335	723	33	
	5	5.5	339	740	27	
	6	4.8	343	754	24	
175	7	4.6	346	766	23	
	8	4.7	348	777	23	
	9	5.0	351	787	22	
	10	5.4	353	794	23	
	1	2.1	316	663	171	
	2	6.5	324	686	63	
	3	7.1	332	707	44	
	4	6.4	337	726	33	
450	5	5.7	342	741	27	
150	6	5.4	345	756	24	
	7	5.4	348	768	23	
	8	5.6	350	779	23	
	9	6.4	357	788	22	
	10	6.5	354	796	23	
	1	7.5	334	697	166	
	2	9.0	338	709	64	
	3	8.4	343	723	44	
	4	7.1	346	737	33	
	5	6.2	349	751	27	
125	6	5.8	352	763	24	
	7	5.8	354	775	23	
	8	6.0	355	785	23	
	9	6.4	357	793	22	
	10	6.8	358	801	23	
	1	10.0	342	717	158	
	2	9.7	343	719	64	
	3	8.6	345	728	44	
	4	7.4	348	740	33	
46-	5	6.7	350	752	27	
100	6	6.3	352	764	24	
	7	6.3	353	775	23	
	8	6.5	355	785	23	
	9	6.8	356	793	22	
	10	7.1	358	800	23	

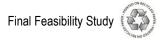


Table A-12 Cross-validation Error Statistics for the LDW-wide Interpolation of cPAHs (continued)

Circular Search					
Radius (ft)	Power	Mean Errora	Mean Absolute Errorb	RMSE	Observed RMSE
	1	4.1	345	714	133
	2	5.7	347	725	63
	3	6.7	350	737	44
	4	7.1	353	749	33
75	5	7.2	355	761	27
75	6	7.3	356	771	24
	7	7.5	358	781	23
	8	7.7	359	790	23
	9	7.9	360	797	22
	10	8.2	361	804	23

- 1. Minimum of 1 and maximum of 10 "nearest neighbor" samples used in all interpolations.
- 2. cPAH interpolation error units in micrograms toxic equivalent per kilograms dry weight (µg TEQ/kg dw).
- 3. Sample count is 828.
- a. Mean Error: The average difference between the observed sample location and the interpolated value at the same location with the sample removed when computing the interpolated value.
- b. Mean Absolute Error: The average of the absolute value of the difference between the observed sample location and the interpolated value at the same location with the sample removed when computing the interpolated value.

Shading within table identifies recommended parameters.

cPAH = carcinogenic polycyclic aromatic hydrocarbon; ft = feet; LDW = Lower Duwamish Waterway; RMSE = root mean square error

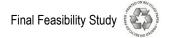


Table A-13 Summary of LDW-wide Observed False Predictions for cPAHs by Concentration Range

	P5/R150 <sup>a</sup>		P5/R175		P6/R150		P6/R175	
cPAH Range (μg TEQ/kg dw)	False Positives	False Negatives	False Positives	False Negatives	False Positives	False Negatives	False Positives	False Negatives
0-90	3	0	3	0	2	0	2	0
>90-150	4	0	4	0	3	0	3	0
>150-380	4	0	4	0	3	0	3	0
>380-900	0	0	0	0	0	0	0	0
>900	0	0	0	0	0	0	0	0
Total False Predictions	1	1	,	11		8		8

- 1. Prediction accuracy is based on comparing the predicted grid-cell value with the concentration range that the empirical value falls within.
- a. Power and search radius parameter combination (e.g., P5/R150 = power of 5 and circular search radius of 150 feet)

cPAH = carcinogenic polycyclic aromatic hydrocarbon; LDW = Lower Duwamish Waterway; µg TEQ /kg dw = micrograms toxic equivalent per kilogram dry weight; P = power; R = radius



Table A-14 Estimated Sediment Surface Areas by Concentration Range for cPAHs

	P5/R150a		F	P5/R175a		P6/R150a		P6/R175a	
cPAH Concentration Range (μg TEQ/kg dw)	Area (acres)	Cumulative Area (acres)							
≤90	69	69	65	65	71	71	67	67	
>90-150	55	124	56	121	55	126	56	122	
>150-380	157	281	159	279	156	282	158	281	
>380-900	121	402	123	402	119	402	121	402	
>900	28	430	27	430	28	430	28	430	

SWAC (µg TEQ/kg dw)	378	376	377
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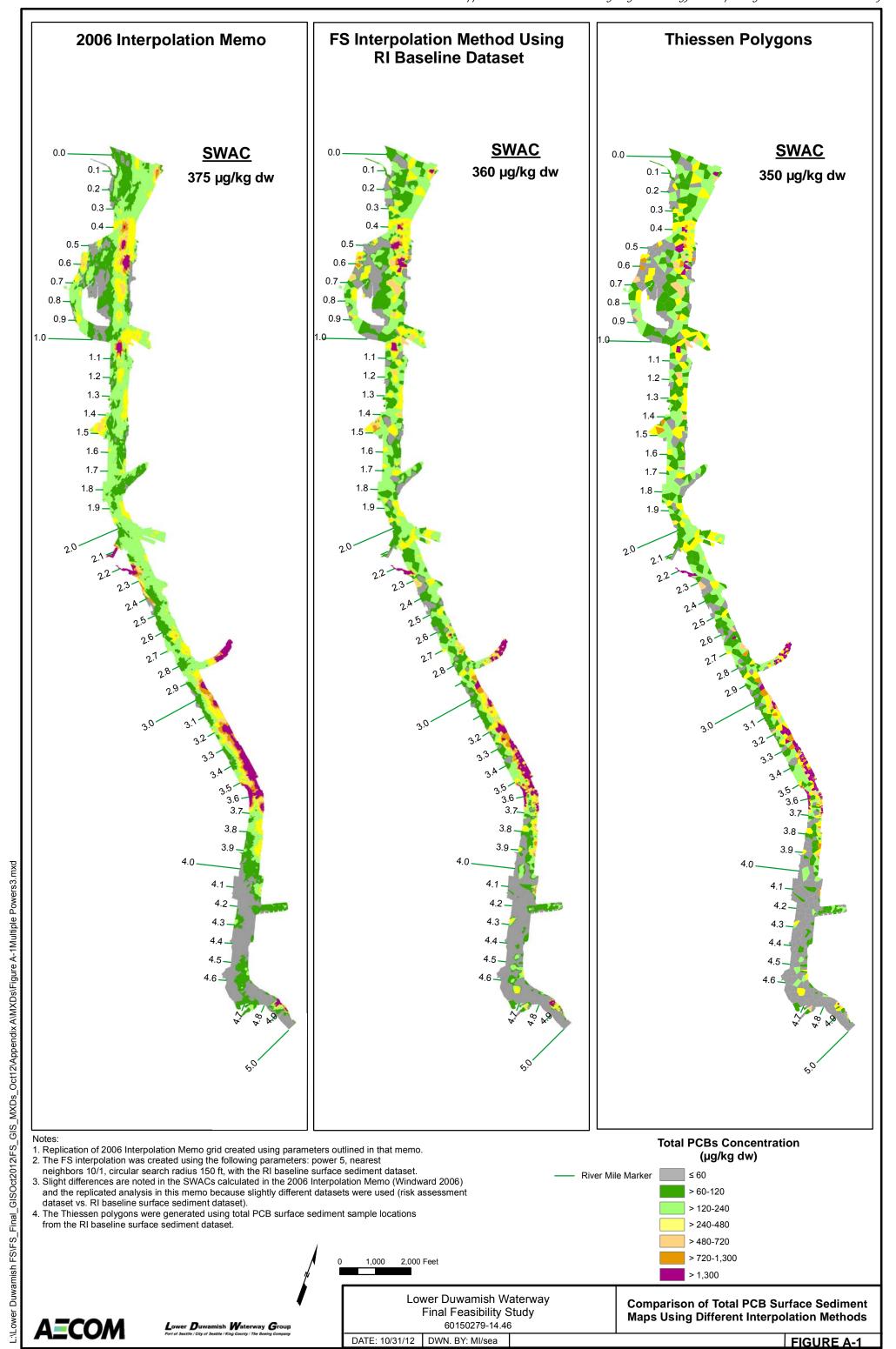
cPAH = carcinogenic polycyclic aromatic hydrocarbon; LDW = Lower Duwamish Waterway; P = power; µg TEQ /kg dw = micrograms toxic equivalent per kilogram dry weight; R = radius; SWAC = spatially-weighted average concentration

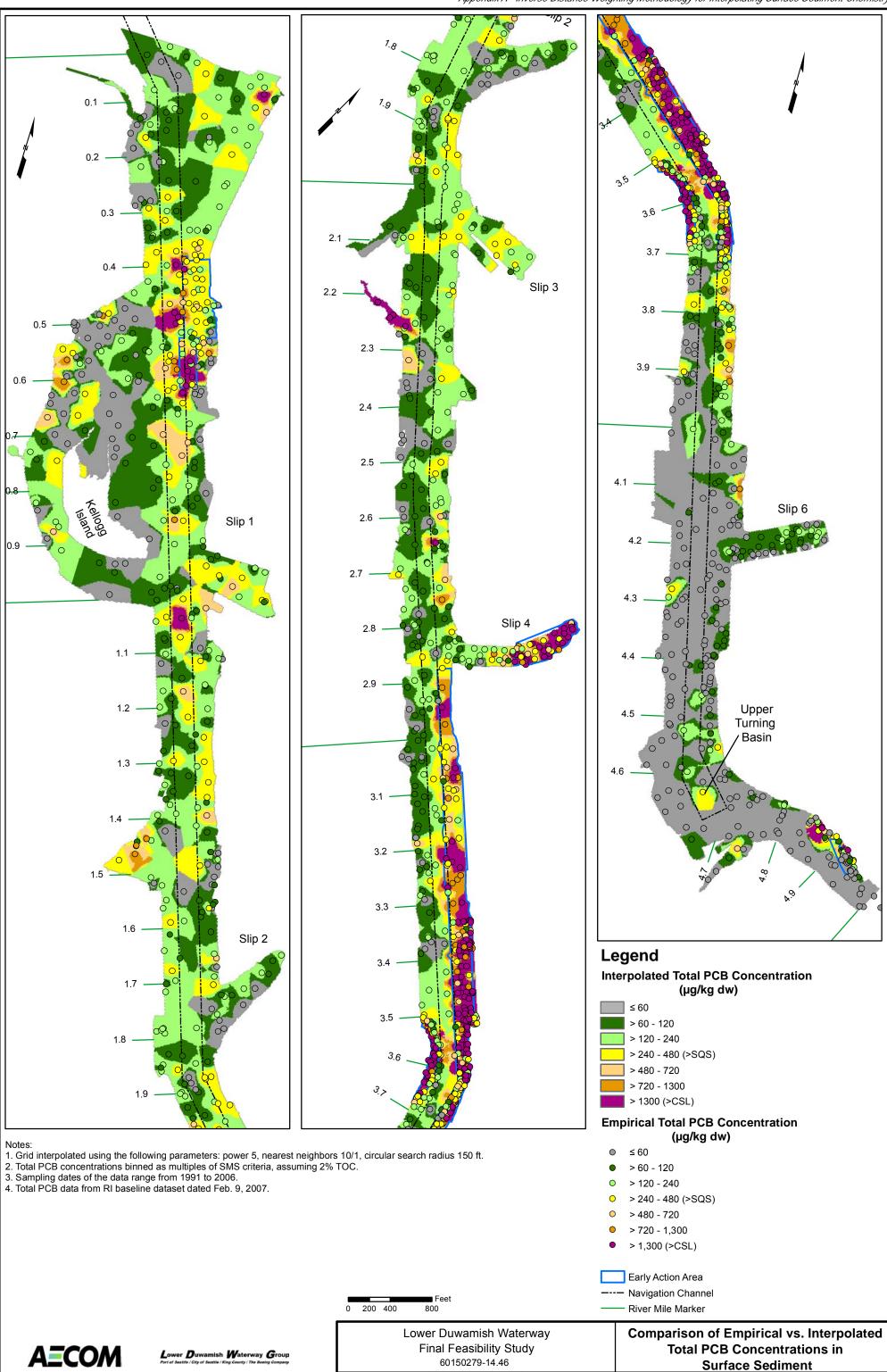




<sup>1.</sup> Total LDW surface area was 430 acres when this analysis was conducted.

a. Power and search radius parameter combination (e.g., P5/R150 = power of 5 and circular search radius of 150 feet)





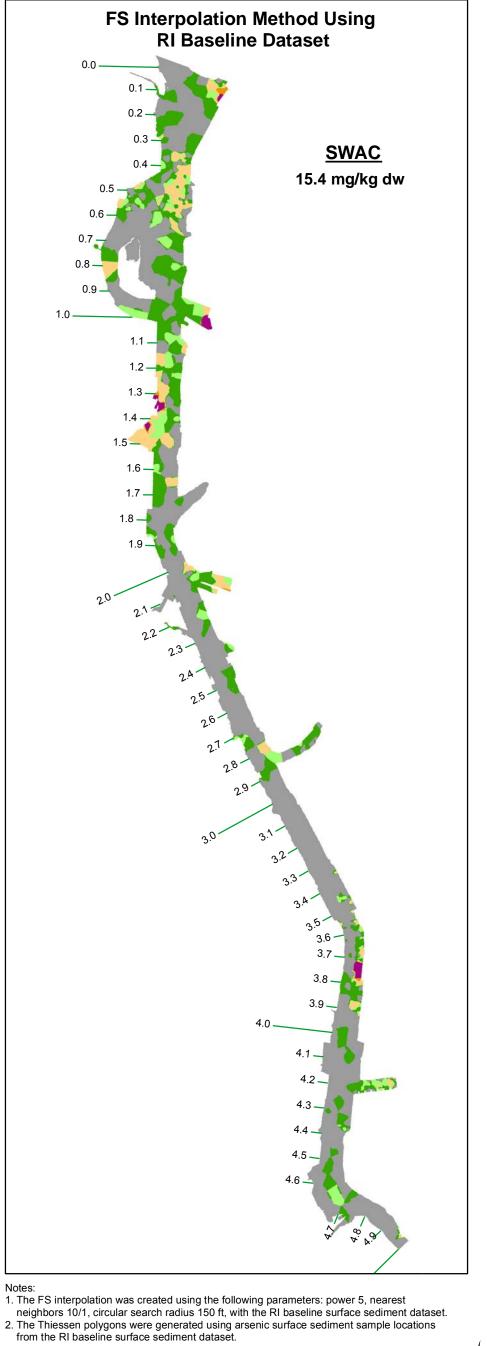
DWRN:MVI/sea

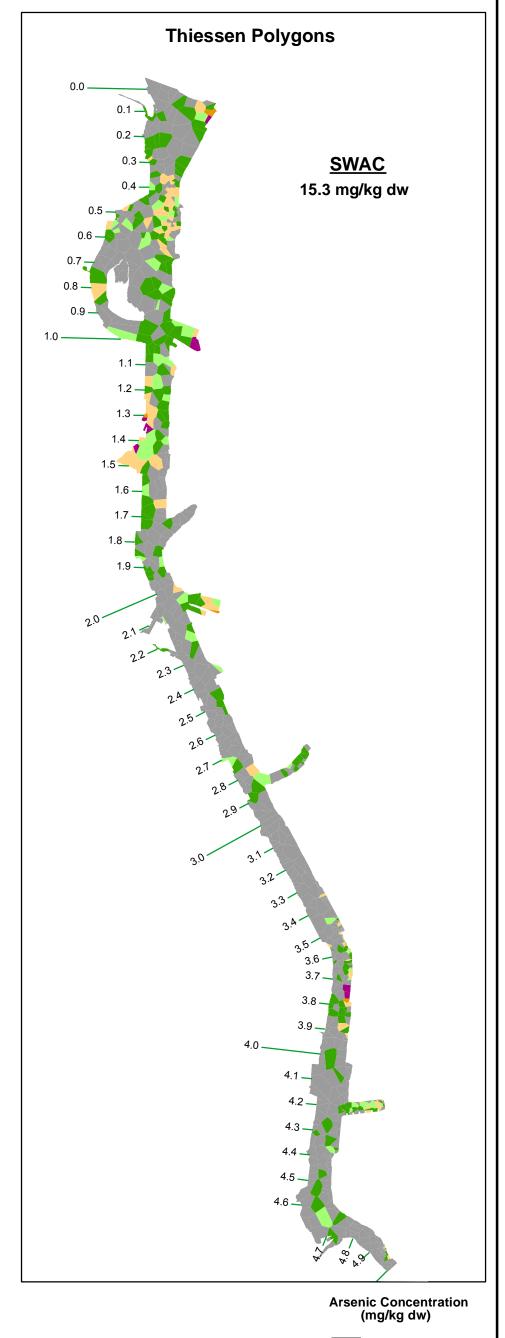
Revision: 0

DATE: 10/31/12

1 - *27* 

**FIGURE A-2** 







≤ 12 River Mile Marker >12 - 16 >16 - 20 >20 - 57 >57 - 93 2,000 Feet > 93

**AECOM** 

Lower Duwamish Waterway Final Feasibility Study 60150279-14.46 DATE: 10/31/12 DWN. BY: MI/sea

**Comparison of Arsenic Surface Sediment** Maps Using Thiessen Polygons and IDW Interpolation

**FIGURE A-3** 

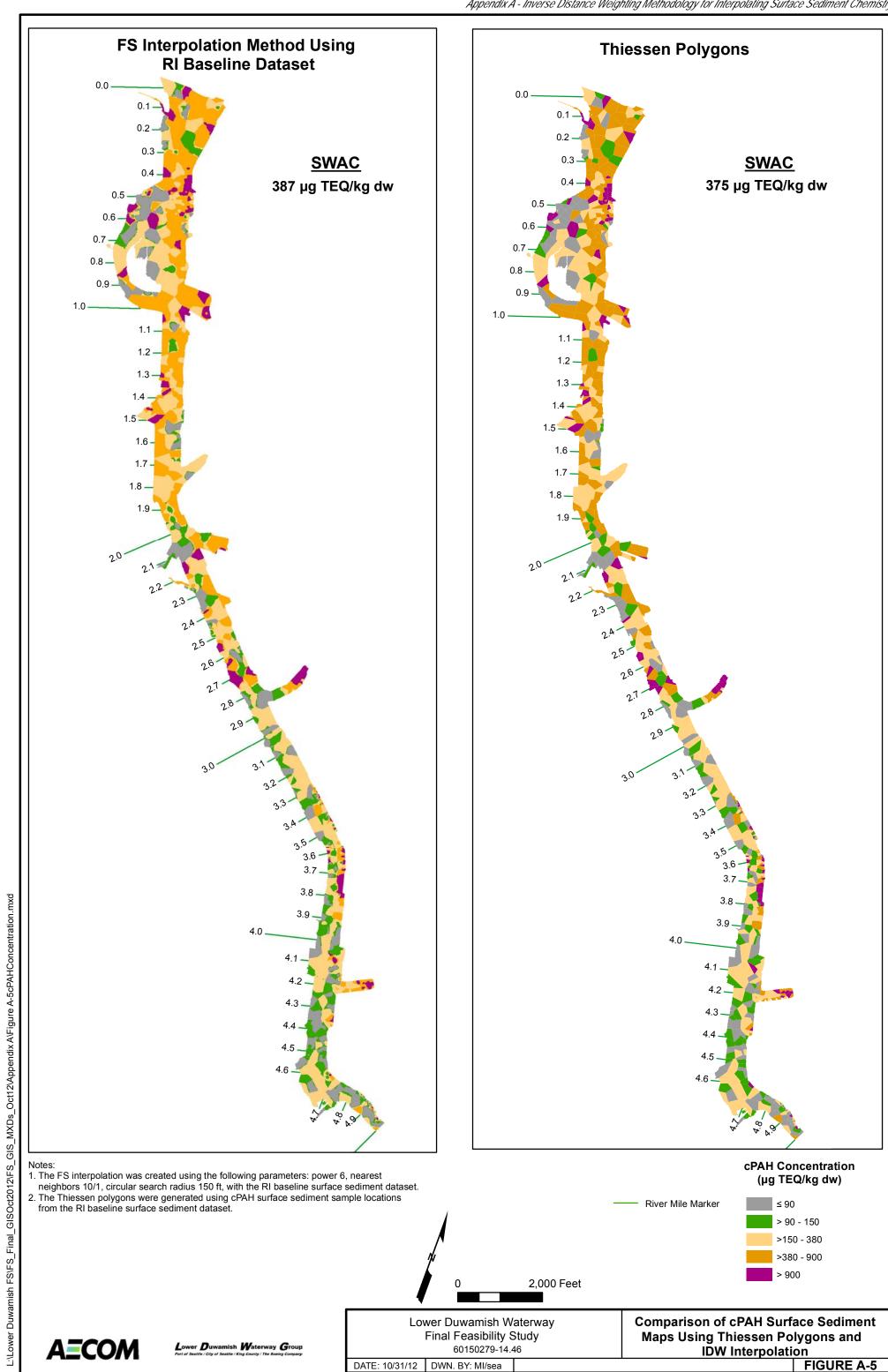


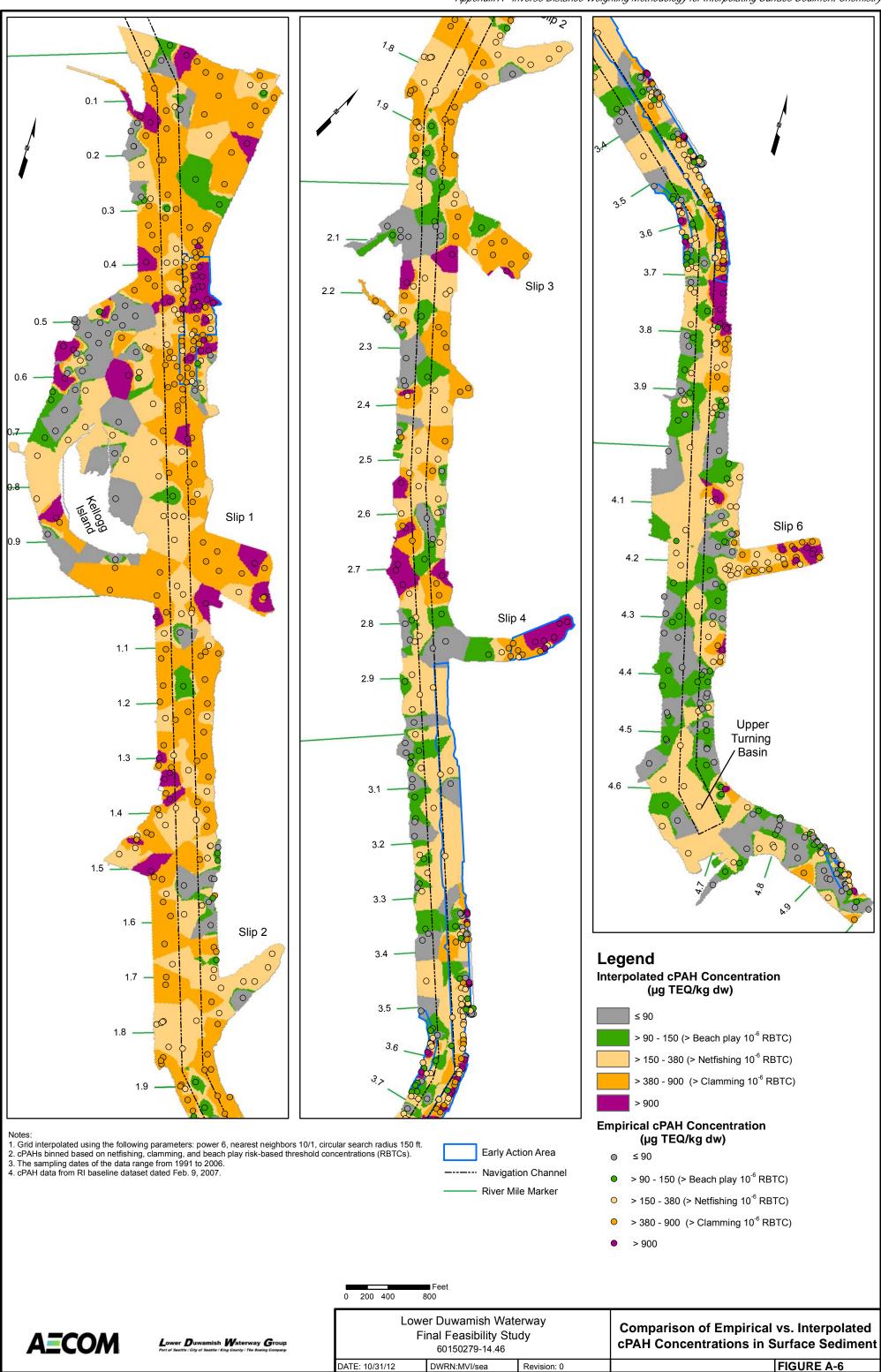
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DATE: 10/31/12

FIGURE A-4





A - 3

## **Attachment A-1**

**The Inverse Distance Weighting Method** 



## The Inverse Distance Weighting Method

The Inverse Distance Weighting (IDW) interpolator within the geographic information systems (GIS) operates on the assumption that entities in close proximity to one another are more alike than those farther away. IDW uses the values of surrounding measured locations to predict the value of unmeasured locations. The measured values closest to the prediction location will have a larger impact on the predicted value than those farther away. Thus, IDW interpolation weights measured values at locations closer to a prediction location more than those farther away.

## The General Formula (esri 2003)

The general formula is:

$$\widehat{Z}(s_o) = \sum_{i=1}^{N} \lambda_i Z(s_i)$$

where:

 $\hat{Z}(s_0)$  is the value we are trying to predict for location  $s_0$ .

*N* is the number of measured sample points surrounding the prediction location that will be used in the prediction.

 $\lambda_i$  are the weights assigned to each measured point that we are going to use. These weights will decrease with distance.

 $Z(s_i)$  is the observed value at the location  $s_i$ .

The formula to determine the weights is the following:

$$\lambda_{i} = d_{i0}^{-p} / \sum_{i=1}^{N} d_{i0}^{-p}$$
  $\sum_{i=1}^{N} \lambda_{i} = 1$ 

As the distance becomes larger, the weight is reduced by a factor of p.

The quantity  $d_{i0}$  is the distance between the prediction location,  $s_0$ , and each of the measured locations,  $s_i$ .

## Parameters that Influence IDW Interpolations

The two primary parameters that affect interpolation methods are exponential power and search radius. These are discussed below.

### **Exponential Power**

The power parameter (p) controls how the weighting factor decreases with distance from a measured location. Weights are proportional to the inverse distance raised to the power p. The greater the power, the less effect distant points have on the value for a predicted location. As a result, the predicted location's value nears the value of the closest point. The converse is also true.

## The Search Neighborhood

The search neighborhood also has a significant impact on the resultant interpolation and acts to limit the extent of the data used to determine the unknown location's value. The search neighborhood has three major components: search radius, shape, and minimum/maximum number of neighbors. The search radius is used to limit the distance from an unknown location that the interpolation method can extend in search of known values. This is done, in part, to improve processing speeds. Also, the similarity of measured values to interpolated point values is expected to diminish with distance.

The shape of the search radius is influenced by the available data and the surface to be created. If there are no discernable directional influences on the weighting of the data, the shape of the search radius should be a circle to consider known sample locations equally in all directions. If there is a directional influence in the data, it can be accounted for by adjusting the shape of the search radius to account for the directionality within the dataset.

When choosing the number of neighbors (minimum/maximum) used for interpolating the value of an unknown location, it is important to consider enough points to yield a good prediction, and few enough points to be practical.

The minimum parameter ensures that at least that specified number of neighbors is used for interpolating the unknown value. The maximum parameter places an upper limit on the number of (nearest) neighboring and measured sample locations used to interpolate the unknown value.

#### References

esri 2003. ArcGIS GeoStatistical Analyst Documentation.