Appendix D. Propeller Wash and Ship Wake Bed Scour Model

A summary of the relevant equations used to calculate bed shear stress is presented here. Detailed descriptions of the ship wake and propeller wash velocity equations are found in Chapter 4 (Velocity Under Shallow Draft Barges) of *Physical Forces near Commercial Tows* (Maynord 2000).

D.1 PROCEDURE FOR CALCULATING BED SHEAR STRESS DISTRIBUTION

The general schematic of a tug-barge moving in a channel, with relevant nomenclature, is shown in Figure D-1.

Total bed shear stress (τ) is given by:

$$\tau = \tau_p + \tau_w \qquad \text{(Equation D-1)}$$

where:

τ_p = bed shear stress due to propeller wash
τ_w = bed shear stress due to ship wake

Note that τ is spatially variable and represents the spatial distribution of bed shear stress in the longitudinal direction behind the ship and in the lateral (cross-channel) direction. This approach incorporates the effects of variable water depth between the navigation channel and bench areas.

Bed shear stress due to propeller wash is calculated using (Maynord 2000):

$$\tau_p = 0.5\ \rho_w\ C_{f,p}\ V^2_{prop} \qquad \text{(Equation D-2)}$$

where:

ρ_w = water density
$C_{f,p}$ = bottom friction factor for propeller wash
V_{prop} = bottom velocity due to propeller wash

The bottom friction factor for propeller wash is given by (Maynord 2000):

$$C_{f,p} = 0.01\ (D_p/H_p) \qquad \text{(Equation D-3)}$$

where:

D_p = propeller diameter
H_p = distance from propeller axis to bottom

Bed shear stress due to ship wake is calculated using:

$$\tau_w = \rho_w C_{f,w} V^2_{wake} \qquad \text{(Equation D-4)}$$

where:

$C_{f,w}$ = bottom friction factor for ship wake
V_{wake} = bottom velocity due to ship wake

The ship wake behaves like a solitary wave as it propagates from the moving ship. The bottom friction factor for a wave depends on the orbital velocity and excursion distance of the wave (van Rijn 1993), which are not determined by the Maynord model. Due to uncertainty in calculating $C_{f,w}$ for a ship wake propagating in a channel with variable bathymetry, the approach proposed in Section 3.2.1 is adopted for use in calculating the bottom friction faction for wake:

$$C_{f,w} = \kappa^2 \ln^{-2} (11 z_{ref}/2D_{90}) \qquad \text{(Equation D-5)}$$

where:

z_{ref} = reference height above the sediment bed
D_{90} = 90th percentile particle diameter of bed sediment
κ = von Karman constant (0.4)

The reference height (z_{ref}) is set equal to half of the local water depth. The value of D_{90} is set at 750 μm, which is average value for the west bench area and it is the maximum D_{90} for the three lateral regions in the LDW. This approach is consistent with the method used to calculate skin friction shear stress in the hydrodynamic and sediment transport modeling studies. It is a reasonable approach when other uncertainties and assumptions used in the analysis are considered.

D.2 Calculation of Bottom Velocity Due to Ship Wake (V_{WAKE})

A coordinate system is fixed to the moving ship. The longitudinal direction (X_p) is aligned with the direction of ship travel, with $X_p = 0$ at the propeller. The lateral direction (Y_{cl}) is aligned with the centerline of the ship. See Figures D-1 and D-2 for definition of various parameters.

In the region from the ship's bow to its stern (i.e., $-(L_{tb} - L_{set}) < X_p < L_{set}$):

$$V_{wake} (X_p) = [(X_p + L_{tb} - L_{set})/ L_{tb}]\ {}^{max}V_{wake} \qquad \text{(Equation D-6)}$$

Behind the ship stern (i.e., $X_p > L_{set}$):

$$V_{wake}(X_p) = [1 - 0.0075(X_p - L_{set})/d_s]\ {}^{max}V_{wake} \quad \text{(Equation D-7)}$$

where:

L_{tb} = length of tugboat
L_{set} = distance from ship stern to propeller
d_s = ship draft

The maximum ship wake velocity, which occurs at the ship stern ($X_p = L_{set}$), is:

$${}^{max}V_{wake} = -0.78\ (d_s / h)^{1.81}\ (V_a - V_g) \quad \text{(Equation D-8)}$$

where:

V_a = ambient average channel velocity
V_g = ship speed relative to ground

D.3 CALCULATION OF BOTTOM VELOCITY DUE TO PROPELLER WASH (V_{PROP})

The region behind the ship propeller, which is located at $X_p = 0$, is divided into two zones. The first zone (Zone 1) is where the propeller jet velocity is dominated by central rudder effects and the jets from the two propellers have not merged. Zone 1 extends from the propeller to a distance of about 10 times the propeller diameter (X_p = 10 D_p, where D_p is propeller diameter). The second zone (Zone 2) is represented by a single jet whose maximum velocity is at the surface; this jet decays both laterally and vertically. Zone 2 starts at X_p = 10 D_p and extends behind the ship. The ship has two propellers that are separated by distance W_p. See Figures D-1 and D-2 for definitions of various parameters.

Zone 1: $X_p / D_p < 10$

In this zone, the propeller jets are dominated by central rudder effects and the two jets have not merged. The total bottom velocity distribution is determined by superposition of the jets from the two propellers. The first step in the analysis is to consider the jet from a single propeller.

Maximum jet velocity for a single propeller in Zone 1 (${}^{Z1}V_{max}$):

$${}^{Z1}V_{max}(x_p) = 1.45\ V_2\ (X_p/D_p)^{-0.524} \quad \text{(Equation D-9)}$$

Velocity increase caused by propeller (V_2):

$$V_2 = (1.13 / D_o)\ (T / \rho)^{1/2} \quad \text{(Equation D-10)}$$

where:

T = propeller thrust
D_o = D_p ➔ ducted (Kort) propeller
= 0.71 D_p ➔ open-wheel propeller

Estimate propeller thrust using the Toutant equations:

Open wheel: $EP = 23.57\ P_{hp}^{0.974} - 2.3\ V_w^2\ P_{hp}^{0.5}$ (Equation D-11)

Kort: $EP = 31.82\ P_{hp}^{0.974} - 5.4\ V_w^2\ P_{hp}^{0.5}$ (Equation D-12)

where:

EP = effective push (equivalent to thrust) from both propellers (lbs)
P_{hp} = total ship power (hp)
V_w = ship speed relative to water (mph)

$V_w = |V_a - V_g|$ (Equation D-13)

Vertical distance from propeller shaft to location of maximum velocity within jet (C_j):

$C_j = [0.213 - 1.05\ (C_p g/V_2^2)\ (X_p - 0.5\ L_{set})]\ (X_p - 0.5\ L_{set})$ (Equation D-14)

where:

C_p = $0.12\ (D_p / H_p)^{0.67}$ ➔ open-wheel propeller
= 0.04 ➔ Kort propeller
g = acceleration due to gravity

Note: maximum $C_j = H_p$

Spatial distribution of bottom velocity for single propeller:

$^{Z1}V\ (X_p, Y_{cl}) = 1.45\ V_2\ (X_p/D_p)^{-0.524}\ \mathrm{EXP}[-(15.4\ R^2/X_p^2)]$ (Equation D-15)

where:

Y_{cl} = lateral distance from ship centerline

$R^2 = (Y_{cl} - 0.5\ W_p)^2 + (H_p - C_j)^2$ (Equation D-16)

where:

W_p = distance between twin propellers

Note: H_p can vary across the transect due to variations in cross-sectional bathymetry, i.e., $H_p = f(Y_{cl})$

Now, superpose bottom velocity distribution from two propellers. Note that this distribution is symmetric about the ship centerline. See Figure D-2.

Total bottom velocity due to propeller wash in Zone 1:

$$^{Z1}V(X_p\ , Y_{cl}) = 1.45\ V_2\ (X_p/D_p)^{-0.524}\{EXP[-(15.4\ R_1^2/\ X_p^2)] + EXP[-(15.4\ R_2^2/\ X_p^2)]\}$$ (Equation D-17)

where:

$R_1^2 = (Y_{cl} – 0.5\ W_p)^2 + (H_p – C_j)^2$ (Equation D-18)

$R_2^2 = (Y_{cl} + 0.5\ W_p)^2 + (H_p – C_j)^2$ (Equation D-19)

Zone 2: Xp / Dp > 10

Zone 2 is represented by a single jet whose maximum velocity is at the surface.

Bottom velocity distribution in Zone 2:

$$^{Z2}V(X_p,Y_{cl}) = 0.34\ V_2\ (D_p/H_p)^{0.93}\ (X_p/D_p)^{0.24}\ C_1\ EXP[-0.0178\ X_p\ /D_p – Y_{cl}^2\ /(2\ C_{z2}\ X_p^2)]$$ (Equation D-20)

where:

C_1 = 0.66 ➔ open-wheel propeller
= 0.85 ➔ Kort propeller

$C_{z2} = 0.84\ (X_p\ /D_p)^{-0.62}$ (Equation D-21)

Thus,

$V_{prop}\ (X_p,Y_{cl})$ = ^{Z1}V, Eq. (D-15), for $X_p\ /D_p < 10$ (Equation D-22)
= ^{Z2}V, Eq. (D-17), for $X_p\ /D_p > 10$

D.4 ESTIMATION OF BED SCOUR DUE TO PASSAGE OF A SINGLE SHIP AT A PARTICULAR LDW TRANSECT

Assume that bow of ship crosses transect RM at time $T_{RM} = 0$. Transform shear stress/velocity distributions from $X_p – Y_{cl}$ plane to $T_{RM} – Y_{cl}$ plane, so that time-variable bottom velocity and shear stress may be calculated. See Figure D-3 for description of parameters. Now,

$X_p = V_gT_{RM} – (L_{tb} – L_{set})$ (Equation D-23)

Time-variable bottom velocity due to the ship wake is determined using Equations D-24 and D-25.

For $0 < T_{RM} < L_{tb} / V_g$ (period when ship is crossing transect):

$$V_{wake}(T_{RM}) = (V_g T_{RM} / L_{tb})\ {}^{max}V_{wake} \qquad \text{(Equation D-24)}$$

For $T_{RM} > L_{tb} / V_g$ (period after ship stern has crossed transect):

$$V_{wake}(T_{RM}) = [1 - 0.0075(V_g T_{RM} - L_{tb})/d_s]\ {}^{max}V_{wake} \qquad \text{(Equation D-25)}$$

Time-variable bottom velocity due to propeller wash is determined using Equations D-26 through D-29.

Zone 1: (Ltb – Lset)/Vg < TRM < (10 Dp + Ltb – Lset)/ Vg

$$^{Z1}V(T_{RM}, Y_{cl}) = 1.45\ V_2\ \{[V_g T_{RM} - (L_{tb} - L_{set})]/D_p\}^{-0.524}$$
$$\{EXP[-15.4\ R_1^2/\ [V_g T_{RM} - (L_{tb} - L_{set})]^2] + EXP[-15.4\ R_2^2/\ [V_g T_{RM} - (L_{tb} - L_{set})]^2]\}$$

(Equation D-26)

and:

$$C_j = [0.213 - 1.05\ (C_p g/V_2^2)\ (V_g T_{RM} - L_{tb} + 0.5\ L_{set})]\ (V_g T_{RM} - L_{tb} + 0.5\ L_{set})$$

(Equation D-27)

Zone 2: TRM > (10 Dp + Ltb – Lset)/ Vg

$$^{Z2}V(T_{RM}, Y_{cl}) = 0.34\ V_2\ (D_p/H_p)^{0.93}\ \{[V_g T_{RM} - (L_{tb} - L_{set})]/D_p\}^{0.24}\ C_1$$
$$EXP\{-0.0178\ [V_g T_{RM} - (L_{tb} - L_{set})]/D_p - Y_{cl}^2\ /(2\ C_{z2}\ [V_g T_{RM} - (L_{tb} - L_{set})]^2)\}$$

(Equation D-28)

and:

$$C_{z2} = 0.84\ \{[V_g T_{RM} - (L_{tb} - L_{set})]/D_p\}^{-0.62} \qquad \text{(Equation D-29)}$$

Use Equations D-24 through D-29 to calculate $\tau(T_{RM}, Y_{cl})$ at transect RM as the ship moves past that location.

For the period that $\tau > \tau_{crit}$ as the ship passes (i.e., $T_o < T_{RM} < T_1$), calculate total bed erosion depth (D_{total}) at various locations across the channel:

$$D_{total}(Y_{cl}) = \int E_{gross}(T, Y_{cl})\ dT \qquad \text{(Equation D-30)}$$

where:

$$E_{gross}(T, Y_{cl}) = A\ [\tau(T_{RM}, Y_{cl})]^n,\ \tau > \tau_{cr} \qquad \text{(Equation D-31)}$$

Sedflume data are used to specify site-specific values of τ_{cr} (critical shear stress), A and n in Equation D-31.

D.5 ESTIMATION OF VARIABLE DEPTH EFFECTS OF WAKE VELOCITY

The maximum wake velocity ${}^{max}V_{wake}$ depends on water depth (h), which is assumed to be constant. Applying the Maynord model to the LDW requires incorporation of variable water depth, in the cross-channel direction, into the calculation of ${}^{max}V_{wake}$ due to significant differences in water depth between the navigation channel and bench areas. An approach, based on shallow water wave theory, for estimating the effects of variable bathymetry on wake velocity is presented here.

For a shallow-water wave, the ratio between the deepwater wavelength (L_o) and the shallow-water wavelength (L) is:

$$L/L_o = (2\pi h/L_o)^{1/2} \quad \text{(Equation D-32)}$$

Similarly, for wave amplitude (A):

$$A/A_o = (8\pi h/L_o)^{-1/4} \quad \text{(Equation D-33)}$$

where the subscript o denotes deepwater conditions.

The maximum horizontal wave velocity at the sediment bed, for a shallow-water wave, is:

$$V = g\ A\ T/[L \cosh(2\pi h/L)] \quad \text{(Equation D-34)}$$

where:

T = wave period (which is constant)

Thus, the ratio of bed wave velocity at two locations with different water depths (i.e., h_1 and h_2) is:

$$V_1/V_2 = (A_1/A_2)\ (L_2/L_1)\ [\cosh(2\pi h_2/L_2)/\cosh(2\pi h_1/L_1)] \quad \text{(Equation D-35)}$$

Using Equations D-32 and D-33:

(L2/L1) = (h2/h1)1/2 (Equation D-36)

(A1/A2) = (h1/h2)-1/4 = (h2/h1)1/4 (Equation D-37)

For shallow-water waves, $h \ll L$ and $\cosh(2\pi h/L) \cong 1$. Thus,

$$V_1/V_2 = (h_2/h_1)^{3/4} \quad \text{(Equation D-38)}$$

Cross-channel variations in maximum wake velocity, due to variable bathymetry in the cross-channel direction, is accomplished in two steps. The first step is to calculate ${}^{max}V_{wake,ave}$ based on the average cross-channel depth (h_{ave}) using Equation D-8. The second step is to determine the local maximum wake velocity (${}^{max}V_{wake,loc}$) using the Equation D-38 and the local depth (h_{loc}):

$${}^{max}V_{wake,loc} = (h_{ave}/h_{loc})^{3/4}\ {}^{max}V_{wake,ave} \quad \text{(Equation D-39)}$$

D.6 GLOSSARY OF TERMS

τ_p	= bed shear stress due to propeller wash (Pa)	
τ_w	= bed shear stress due to ship wake (Pa)	
ρ_w	= water density (g/cm^3)	
$C_{f,p}$	= bottom friction factor for propeller wash (unitless coefficient)	
V_{prop}	= bottom velocity due to propeller wash (m/sec)	
D_p	= propeller diameter (m)	
H_p	= distance from propeller axis to bottom (m)	
V_{wake}	= bottom velocity due to ship wake (m/sec)	
$C_{f,w}$	= bottom friction factor for ship wake (unitless coefficient)	
z_{ref}	= reference height above the sediment bed (m)	
D_{90}	= 90th percentile particle diameter of bed sediment (μm)	
κ	= von Karman constant (0.4)	
L_{tb}	= length of tugboat (m)	
L_{set}	= distance from ship stern to propeller (m)	
d_s	= ship draft (m)	
V_a	= ambient average channel velocity (m/sec)	
V_g	= ship speed relative to ground (m/sec)	
T	= propeller thrust (Newtons – N)	
D_o	= D_p	➔ ducted (Kort) propeller
	= 0.71 D_p	➔ open-wheel propeller
EP	= effective push (equivalent to thrust) from both propellers (lbs)	
P_{hp}	= total ship power (hp)	
V_w	= ship speed relative to water (mph)	
C_p	= $0.12\,(D_p / H_p)^{0.67}$	➔ open-wheel propeller
	= 0.04	➔ Kort propeller
g	= acceleration due to gravity (m/s^2)	
Y_{cl}	= lateral distance from ship centerline (m)	
W_p	= distance between twin propellers (m)	
C_1	= 0.66	➔ open-wheel propeller
	= 0.85	➔ Kort propeller
T	= wave period (s)	
L	= wave length (m)	
A	= wave amplitude (m)	

D.7 REFERENCES

Maynord ST. 2000. Physical forces near commercial tows. Interim report for the Upper Mississippi River-Illinois Waterway System Navigation Study. Env Report 19, interim report. US Army Corps of Engineers Research and Development Center, Vicksburg, MS.

List of Figures Cited in Appendix D

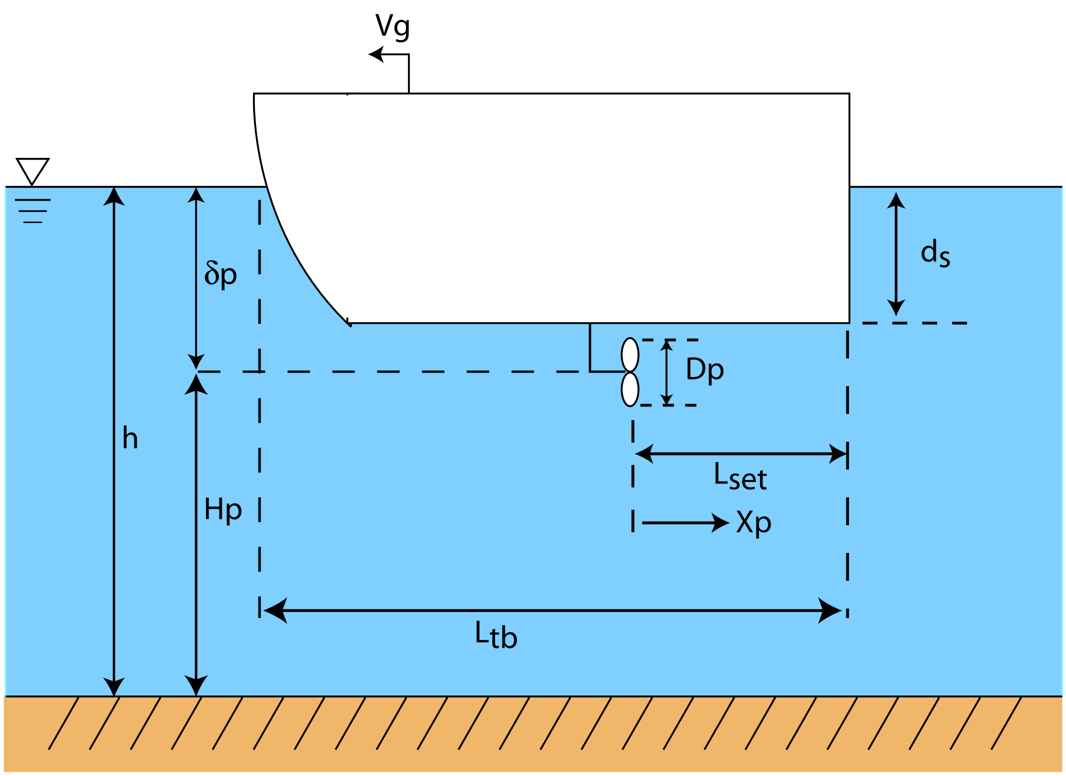

Figure D-1. Nomenclature for ship geometry used in propeller wash

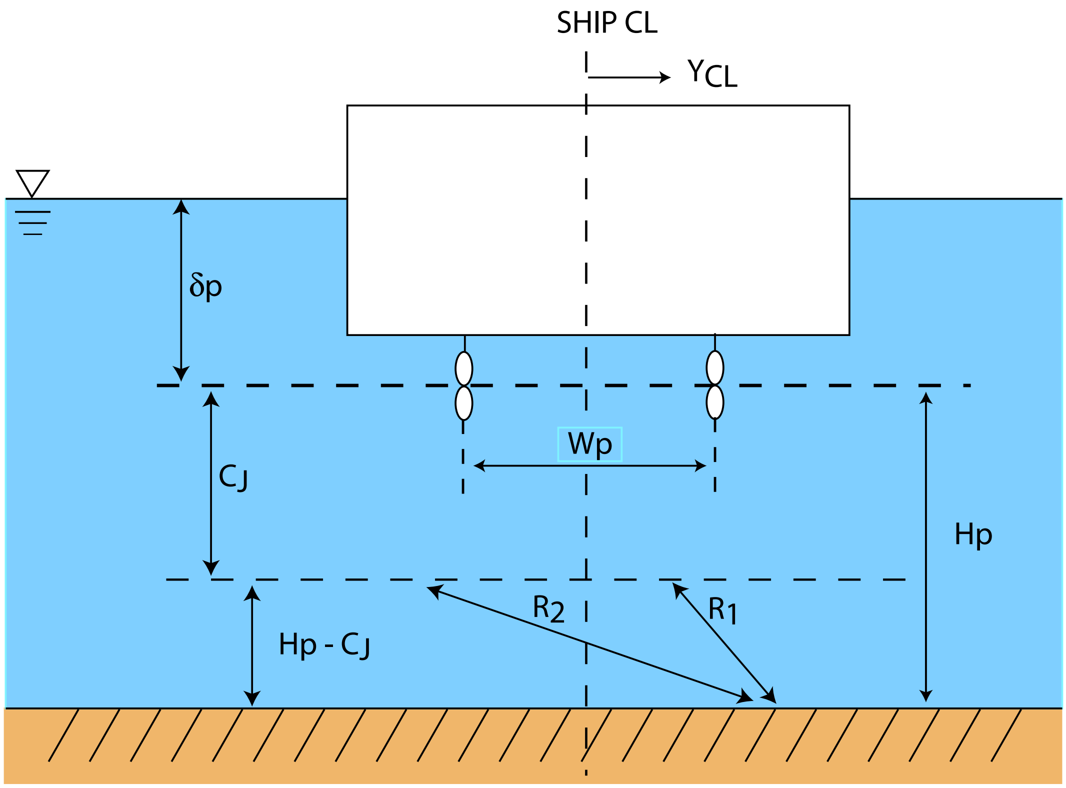

Figure D-2. Nomenclature for parameters used in propeller wash model.

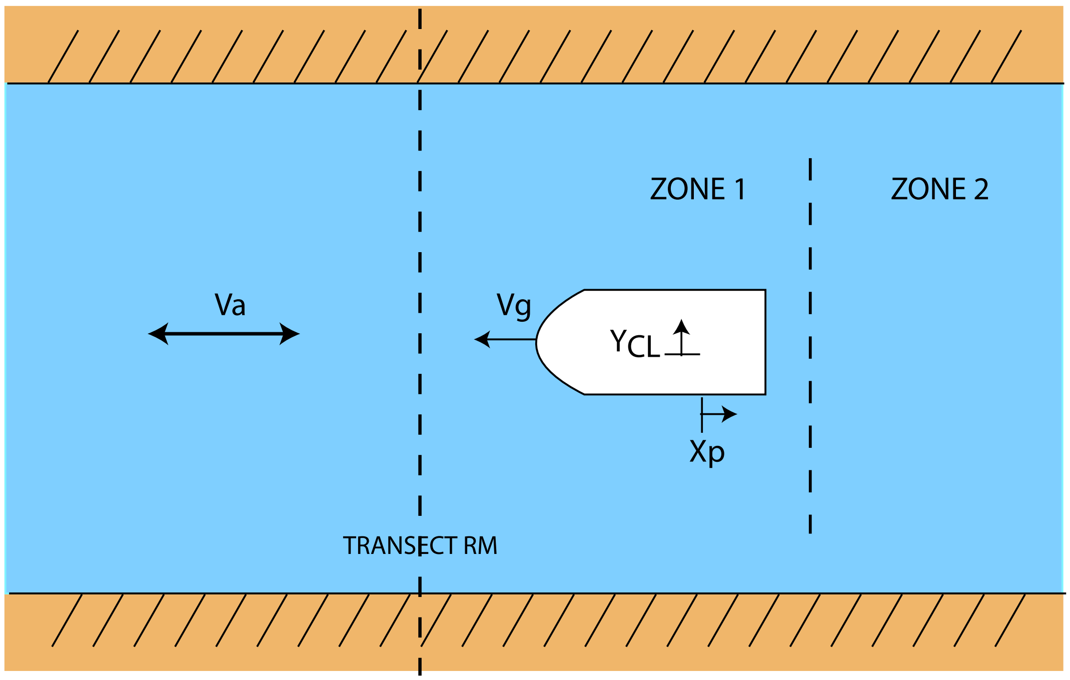

Figure D-3. Nomenclature used to define movement of ship past a transect.